

**Exercise 47**

Find the limit.

$$\lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{2\theta^2}$$

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**Solution**

Rewrite the limit in terms of one that is known.

$$\begin{aligned} \lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{2\theta^2} &= \lim_{x \rightarrow 0} \frac{\cos 2x - 1}{2(2x)^2} = \lim_{x \rightarrow 0} \frac{-2 \sin^2 x}{8x^2} = -\frac{1}{4} \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2} \\ &= -\frac{1}{4} \left( \lim_{x \rightarrow 0} \frac{\sin x}{x} \right) \left( \lim_{x \rightarrow 0} \frac{\sin x}{x} \right) \\ &= -\frac{1}{4}(1)(1) \\ &= -\frac{1}{4} \end{aligned}$$